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NAVAL POSTGRADUATE SCHOOL
Monterey, California

A STOCHASTIC MODEL OF A REPAIRABLE
ITEM INVENTORY SYSTEM

by

F. Russell Richards

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NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral M. B. Freeman, USN
Superintendent

M. U. Clauser
Provost

ABSTRACT:

An inventory system for repairable items is studied. A stochastic model which coordinates procurement and repair decisions is developed. Special attention is paid to modeling the repairable item system so that the results derived in this report are applicable for Navy inventory management. Long-run distributions for both the ready-for-issue and the non-ready-for-issue stock and many useful measures of performance are determined. Uses of the information to analyze the critical factors in a repairable item system and to determine optimal values of the parameters are pointed out. Numerical examples of the calculations of the measures of effectiveness are presented. Finally, modifications of the model to include the zero attrition case and consumable items are pointed out.

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Prepared by:

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1. INTRODUCTION

The Naval Supply System is responsible for stocking a number of items which are failure prone and have been designated as repairable. Because of engineering and economic considerations attempts are made to restore the items to serviceable condition whenever an item fails. Although the repairable items account for only a small percentage of the quantity of items stocked throughout the Navy, they account for quite a large portion of the dollars invested in inventory. The characteristics of high cost, low demand and high essentiality that these items typically possess make efficient inventory control difficult and extremely critical.

Repairable item inventory systems have been investigated in two previous reports ([1] and [2]) by personnel at the Naval Postgraduate School. Other work in the area, primarily by the armed services, is summarized extensively in reference [2]. Both deterministic and probabilistic models of the repairable item inventory system have been examined, but an exact solution of the continuous review stochastic model considered in this report has not previously been achieved.

Special effort has been made to structure the model to be compatible with the Naval Supply System. Random failures, random repair times and positive procurement lead times are all included. Assuming a simple class of policies in which procurement decisions are made with cognizance of the repair situation, we determine stationary distributions for the items in repair, the serviceable stock on hand and the number of backorders. From these distributions we calculate such measures of performance as the probability of a stockout, the total expected backorder days, the expected number of items ready for issue and the mean supply response times.

Numerical examples are presented with comments about several possible uses of the results, such as performing sensitivity analyses and determining optimal choices of parameters and initial stocking quantities.

2. ASSUMPTIONS

Consistent with present Navy usage and previous studies ([3], [4], and [5]) of demand distributions for low demand items, the failure of any unit is assumed to be independent of the condition of any other unit, and the number of units which fail in any period of time is taken to be a realization of a random variable having a Poisson mass function with mean rate λ. To allow for attrition of some of the failed items we assume that each failed unit is inspected to determine if it will enter the repair cycle or be scrapped. The inspections are repeated independent Bernoulli trials with the probability of entering the repair cycle a known value p, and a probability of $1 - p$ that the unit is scrapped. This assumption divides failures into two natural classes: (1) those failures which will be repaired, and (2) those failures which will be scrapped. Denote these two types of failures as Type 1 failures and Type 2 failures, respectively.

In order to reduce the time required to replace a failed unit by a serviceable unit, an attempt is made to maintain a stock of spares on hand. Thus, if a unit is available, the end user is sent a replacement as soon as his need is made known. If a spare is not available when a part fails, the end user must wait until a unit is repaired or a procurement is received. The pool of serviceable spares is composed of units which have been requisitioned as well as those which have been repaired. These two types of serviceable units are assumed to be indistinguishable to the user. Whenever a failure occurs and a spare is not available, a backorder is created. Since some penalty is suffered each time a backorder occurs, it should be the objective of any supply system to minimize the number of backorders subject to available resources.

In order to maintain the pool of spares stock must be supplemented from time to time by the procurement of new units. Hence, an important problem to be faced is that of determining a procurement policy for this replenishment which is fully cognizant of the situation at the repair facility. In this report, no attempt is made to determine "the optimal policy" to be used. Instead, we focus attention on a class of simple policies which is completely specified by two numbers, a reorder quantity and a reorder level. In order to describe this class of policies a few modifications in the definitions commonly used in the literature for consumable item inventory systems must be made.

Definition 2.1: The on-hand inventory includes all of the items ready for issue less any backorders. The on-hand inventory at time t is denoted by $H(t)$, and is negative if and only if backorders exist.

Definition 2.2: The repair inventory includes all of those items at the repair facility. The repair inventory at time t is denoted by $X(t)$.

Definition 2.3: The net inventory includes the on-hand inventory and the repair inventory. Net inventory at time t is denoted by $Y(t)$. Thus, $Y(t) = X(t) + H(t)$.

Definition 2.4: The inventory position is the sum of the net inventory plus all items which have been placed on order but have not yet been received. $I(t)$ represents the inventory position at time t.

Figure 1 reveals the relationships among the net inventory, the repair inventory, the on-hand inventory, and the inventory position.

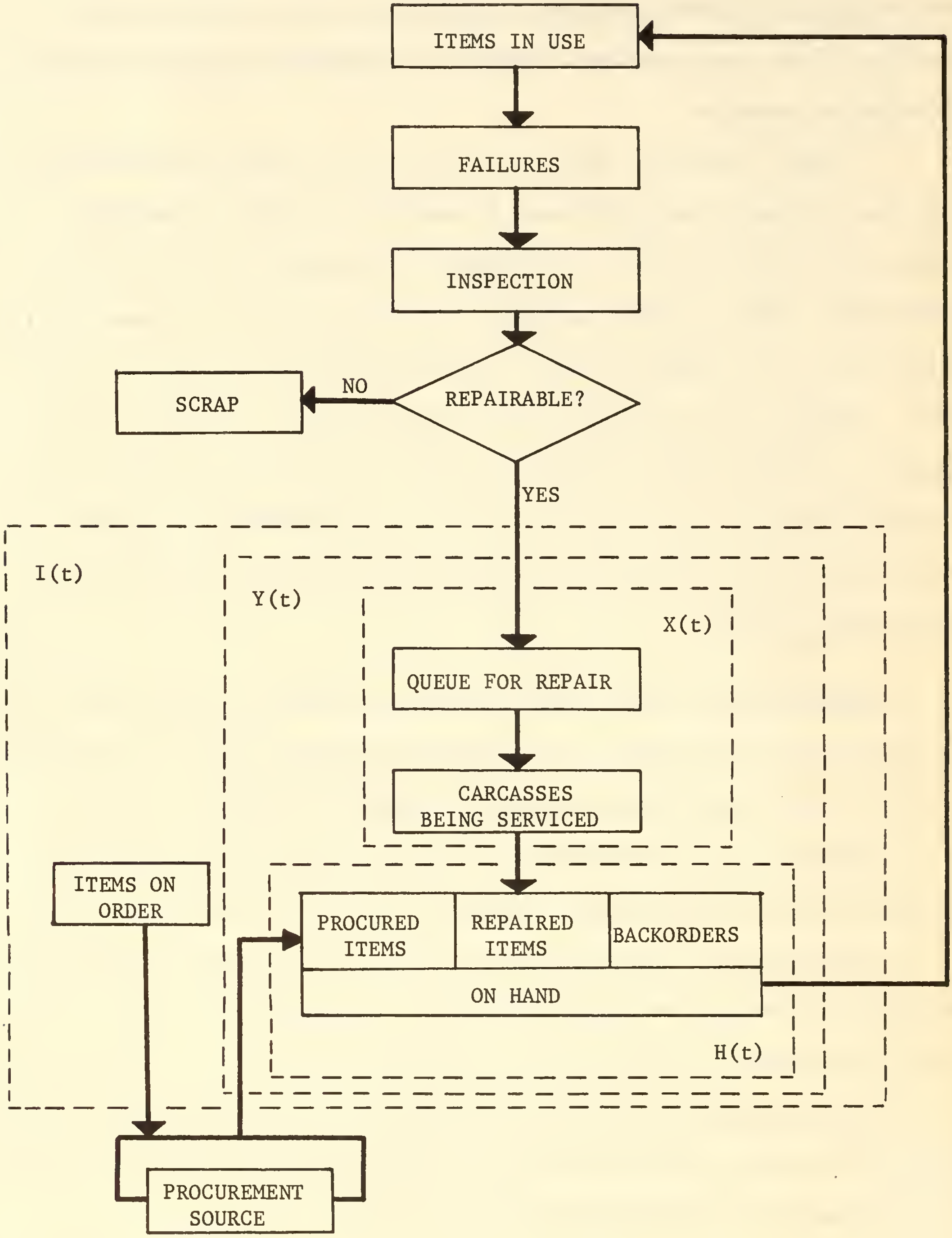

Figure 1. REPAIRABLE ITEM INVENTORY SYSTEM.

We can now describe the class of procurement policies considered in this report. When the inventory position reaches or falls below a fixed level r, a procurement order for Q units is made. We assume that an order is received after a time lag of length L. Inventory replenishment policies of this type are widely used for consumable item systems and have even been shown to be optimal for those systems in the sense that they yield the minimum expected cost per unit time. For the repairable item system these policies are intuitively appealing because of their procedural simplicity. Furthermore, analytical probabilistic evaluations of such policies may be carried out. These evaluations allow us to make predictions and firm quantitative statements about the future operating characteristics of the policies. Lastly, the number of items in repair is made an integral part of the procurement policy. Because of the latter factor a procurement order would not be placed if the repair inventory were high. In fact, it should be noted that, under the specified procurement policy, there may be backorders at some time without a replenishment order being made. For example, such would be the case at time t if $X(t) = r + j + 1$ and $H(t) = - j$ so that $Y(t) = r + 1$. The reorder point r has not yet been hit although there are j backorders. If the inventory system were a consumable item system, backorders would not normally be allowed before a replenishment was made. However, in the repairable item system, one must be cognizant of the repair inventory. If those items at the repair facility were ignored when making replenishment decisions, the total number of items in the inventory system could grow arbitrarily large. This would result in increased ordering costs, stock investment and holding costs. On the other hand, the total number of items in the system is bounded above by

$r + Q$ under the recommended class of policies. Furthermore, it is usually the case for repairable items that the mean repair time is less than the procurement lead time, and it is more economical to repair than it is to procure. Thus, in a case such as the example above, there would probably be a high likelihood that the repair facility will generate serviceable units as quickly as a procurement and at less cost.

The repair policy which we consider assumes that there are ample servers (repair crews) so that a unit would never have to wait in a queue before it is inducted for repair. Therefore, batching of units for repair is not considered. The repair times are assumed to be independent and identically distributed random variables $T_1, T_2, \ldots$ with known distribution function $R(t)$. The only requirement about the repair time distribution is that the mean is finite. This repair policy concentrates on getting units through the repair facility with as short a delay as possible. Additional discussion about the repair assumptions follows in the conclusion of this report.

3. THE INDEPENDENCE OF THE NET INVENTORY AND THE REPAIR INVENTORY

We now derive some results which are of fundamental importance in all the material which follows. Let the process which counts the number of failures that occur in the interval $(0,t]$ be a Poisson process $\{N(t), t \geq 0\}$ with mean rate λ. As described earlier, a failure is classified as a Type 1 failure with probability p or a Type 2 failure with probability $1 - p$. Let $N_1(t)$ be the number of Type 1 failures in the interval $(0,t]$, and let $N_2(t)$ be the number of Type 2 failures in the same interval. The following well-known theorem [6] gives us important information about the stochastic processes $\{N_1(t), t \geq 0\}$ and

$\{N_2(t),\ t \geq 0\}$.

Theorem 3.1: (i) Let $\{N(t),\ t \geq 0\}$ be a Poisson process generating arrivals with rate λ. Suppose that an arrival which occurs at time t is classified, independent of other arrivals, into one of k categories with probability p_i where $\sum_{i=1}^{k} p_i = 1$. If $N_i(t)$ denotes the number of arrivals in the interval $(0,t]$ which belong to the i^{th} category, then $\{N_i(t),\ t \geq 0\}$ is a Poisson process with mean rate $p_i \lambda$ for each $i = 1, 2, \ldots, k$.

(ii) For each $t \geq 0$, the random variables $N_1(t), \ldots, N_k(t)$ are independent.

Theorem 3.1 tells us that the processes which count Type 1 failures and Type 2 failures are Poisson processes with mean rates $p\lambda$ and $(1-p)\lambda$, respectively. Moreover, $N_1(t)$ and $N_2(t)$ are independent for each t. Since a Poisson process has stationary independent increments, the following result has been proved.

Corollary 3.1: The number of Type 1 failures in an arbitrary interval of time is independent of the number of Type 2 failures in the same interval.

We will now show the somewhat surprising result that the net inventory and the repair inventory are independent! Toward that end, we examine the manner in which transitions are made in the two stochastic processes $\{X(t),\ t \geq 0\}$ and $\{Y(t),\ t \geq 0\}$.

Whenever a Type 1 failure occurs $H(t)$ decreases by one, but $X(t)$ increases by one for a net change of zero in $Y(t)$. On the other hand, $H(t)$ decreases by one while $X(t)$ remains constant when a Type 2 failure occurs. Thus, a Type 2 failure causes a net decrease of one in $Y(t)$, but does not affect $X(t)$.

The arrival of a procurement increases $Y(t)$ by the amount Q, while the repair inventory is unaffected. Conversely, $X(t)$ decreases while $H(t)$ increases by a like amount when items leave repair. Thus, repairs decrease $X(t)$ but leave $Y(t)$ unchanged.

Define $F_1(t)$ and $F_2(t)$ to be the total number of items repaired and the total number of items received from procurements, respectively, in the interval $(0,t]$. Then if $C_1 = X(0)$ and $C_2 = Y(0)$, the repair inventory at time t and the net inventory at time t are respectively given by

$$X(t) = C_1 + N_1(t) - F_1(t)$$

and

$$Y(t) = C_2 + F_2(t) - N_2(t).$$

So long as the repair disciplines and the procurement disciplines are as stated, the following observations can be made. As a function of the random variables $N_1(t)$ and $N_2(t)$, $F_1(t)$ depends on $N_1(t)$ and is independent of $N_2(t)$; on the other hand, $F_2(t)$ depends on $N_2(t)$ and is independent of $N_1(t)$. Consequently, as functions of the random variables $N_1(t)$ and $N_2(t)$, $X(t)$ is a function of $N_1(t)$ alone while $Y(t)$ is a function only of $N_2(t)$. We have shown that $N_1(t)$ and $N_2(t)$ are independent for each t. Being functions of independent random variables, $Y(t)$ and $X(t)$ are therefore themselves independent random variables. This key result is stated as the next theorem.

Theorem 3.2: The random variables $X(t)$ and $Y(t)$ are independent for each t.

4. THE DISTRIBUTION OF THE NET INVENTORY

We will now determine the stationary distribution of the net inventory, $Y(t)$. First, we determine the distribution of the inventory position.

Under the prescribed procurement policy the inventory position at time t, $I(t)$, must be between $r + 1$ and $r + Q$. In addition, the state of the process $\{I(t), t \geq 0\}$ only changes when Type 2 failures occur. Thus, the inventory position for this problem, although it includes the repair inventory, acts just as the inventory position of a consumable item problem where the demands correspond to Type 2 failures. Since Theorem 3.1 tells us that Type 2 failures are Poisson distributed, we can use a well known fact [3] about the inventory position in a consumable item system where demands are Poisson distributed to get the following theorem.

Theorem 4.1: The stochastic process $\{I(t), t \geq 0\}$ has a unique stationary distribution, Furthermore, if η_j denotes the stationary probability that the inventory position is $r + j$, then $\eta_j = 1/Q$ for $j = 1, 2, \ldots, Q$.

Since the inventory position in the repairable item system bounces uniformly between $r + 1$ and $r + Q$, the expected value of the inventory position is $r + \frac{Q+1}{2}$. Let us now turn attention to the distribution of the net inventory. Let $f(n; (1-p)\lambda t)$ be the probability of n Type 2 failures in an arbitrary interval of length t; that is,

$$f(n; (1-p)\lambda t) = P[N_2(s+t) - N_2(s) = n].$$

$N_2(t)$ is Poisson distributed with mean $(1-p)\lambda t$. Hence,

$$f(n; (1-p)\lambda t) = \frac{[(1-p)\lambda t]^n e^{-(1-p)\lambda t}}{n!}.$$

Theorem 4.2: The stationary probability mass function of the net inventory is given by:

$$\alpha(x) = \begin{cases} 1/Q \sum_{u=r+1-x}^{r+Q-x} f(u; (1-p)\lambda L) & \text{if } r \geq x \\ 1/Q \sum_{u=0}^{r+Q-x} f(u; (1-p)\lambda L) & \text{if } r+1 \leq x \leq r+Q \\ 0 & \text{if } x > r+Q. \end{cases}$$

Furthermore, if m is the mean number of Type 2 failures in a period of time of length L the expected value of the net inventory is $r + \frac{Q+1}{2} - m$.

Proof: Theorem 4.1 tells us that the inventory position is uniformly distributed on the set $\{r+1, r+2, \ldots, r+Q\}$. Let s be an arbitrary point in time after the inventory system has been in operation a long period of time. Everything on order at time $s - L$ will be available by time s, but any units ordered after time $s - L$ can only arrive later than time s. If the inventory position at time $s - L$ is $r + j$, there will be exactly $r + j$ items available to replace Type 2 failures which occur in the interval $(s-L,s]$. The conditional probability that the net inventory is x, given that the inventory position at time $s - L$ is $r + j$, is simply the probability that the number of Type 2 failures in $(s-L,s]$ is $r + j - x$ if $r + j \geq x$, and the probability is zero if $r + j < x$. Averaging these conditional probabilities over all values of the inventory position at time $s - L$, we obtain for $x \leq r + j$

$$\alpha(x) = \sum_{j=1}^{Q} f(r+j-x;\ (1-p)\lambda L)$$

so that

$$\alpha(x) = \begin{cases} 1/Q \displaystyle\sum_{u=r+1-x}^{r+Q-x} f(u;\ (1-p)\lambda L) & \text{if } x \leq r \\ 1/Q \displaystyle\sum_{u=0}^{r+Q-x} f(u;\ (1-p)\lambda L) & \text{if } r+1 \leq x \leq r+Q \\ 0 & \text{if } x > r+Q. \end{cases} \tag{4.3}$$

The expected value of the net inventory is determined by a straightforward calculation.

$$E[Y] = \sum_{x=-\infty}^{r+Q} x\alpha(x)$$

On substituting equation (4.3) for $\alpha(x)$, interchanging the order of summation, and simplifying, we get

$$E[Y] = (r+\frac{Q+1}{2}) \sum_{j=0}^{\infty} f(j;\ (1-p)\lambda L) - \sum_{j=1}^{\infty} jf(j;\ (1-p)\lambda L)$$

$$= r + \frac{Q+1}{2} - m.$$

This completes the proof of Theorem 4.2.

The reader experienced in the inventory literature may recognize the distribution of the net inventory derived above to be the same as that derived for the net inventory in a consumable item system where demands are Poisson distributed with rate $(1-p)\lambda$. That this is the case should not be surprising in light of the fact that the state of the net inventory changes only when Type 2 failures occur and procurements arrive.

5. THE DISTRIBUTION OF THE REPAIR INVENTORY

We have seen from Theorem 3.1 that the arrivals of failed items at the repair facility are Poisson distributed with mean rate $p\lambda$. We will now determine the distribution of the number of items at the repair facility.

Define a function $w(s,y)$ as follows:

$$w(s,y) = \begin{cases} 1 & \text{if } 0 \leq s < y \\ 0 & \text{otherwise.} \end{cases}$$

A Type 1 failure which enters the repair facility at time τ will still be at the repair facility at time t if and only if the time required to repair that item is greater than $t - \tau$. Consequently, if τ_m denotes the time at which the m^{th} Type 1 failure occurs and T_m is the time required to repair the m^{th} Type 1 failure, the number of units at the repair facility at time t is given by

$$X(t) = \sum_{m=1}^{N_1(t)} w(t-\tau_m,\ T_m).$$

Now $\{N_1(t), t \geq 0\}$ is a Poisson process and $\{T_m\}$ is a sequence of independent random variables identically distributed as a random variable T and independent of the counting process $\{N_1(t), t \geq 0\}$. $\{X(t), t \geq 0\}$ is therefore a filtered Poisson process, and Parzen [7] has shown that the characteristic function of $X(t)$ is given by

$$(5.1) \qquad C_{X(t)}(u) = \exp\left\{p\lambda \int_0^t E[e^{iuw(t-\tau,T)} - 1]\, d\tau\right\}$$

for any positive number t and real number u. Equation (5.1) will be used to prove the following theorem.

<u>Theorem 5.2</u>: Let a Poisson process with mean rate $p\lambda$ generate Type 1 failures, and let $E[T]$ be the mean of the repair time distribution $R(t)$. The limiting distribution of the number of items at the repair facility is Poisson with parameter $p\lambda E[T]$.

Proof: Equation (5.1) gives the characteristic function for the repair inventory at time t. Consider the random variable $\exp(iuw(t-\tau,T)) - 1$.

$$\exp(iuw(t-\tau,T)) - 1 = \begin{cases} e^{iu} - 1 & \text{if } 0 \leq t - \tau < T \\ 0 & \text{otherwise.} \end{cases}$$

Thus,

$$\begin{aligned} E\,[\exp(iuw(t-\tau,T))-1] &= (e^{iu}-1)\, P[T>t-\tau] \\ &= (e^{iu}-1)\, (1-R(t-\tau)). \end{aligned}$$

Hence,

$$C_{X(t)}(u) = \exp\{p\lambda(e^{iu}-1) \int_0^t (1-R(t-\tau))d\tau\}$$

which is the characteristic function of a random variable having a Poisson distribution with mean $p\lambda \int_0^t (1-R(t-\tau))d\tau$, abbreviated by μ_t.

Then,

$$\lim_{t\to\infty} P[X(t)=x] = \lim_{t\to\infty} \frac{e^{-\mu_t}(\mu_t)^x}{x!} \quad .$$

A change of variable gives

$$\lim_{t\to\infty} \mu_t = \lim_{t\to\infty} p\lambda \int_0^t (1-R(s))ds = p\lambda E[T].$$

The last equality follows because the expected value of a non-negative random variable Z with finite mean and distribution $F_Z(z)$ is given by

$$E[Z] = \int_0^\infty (1-F_Z(z))d_z.$$

This gives the desired result:

$$(5.2) \qquad \lim_{t\to\infty} P[X(t)=x] = \frac{e^{-p\lambda E[T]}(p\lambda E[T])^x}{x!} = f(x;\ p\lambda E[T])$$

completing the proof of Theorem 5.2.

A few important observations about the distribution of the repair inventory are in order. First, the limiting distribution depends on the repair time distribution only through its mean value. The form of the distribution itself is unnecessary for one can easily obtain reliable estimates of the mean repair time. Also, it is probably the case in actual practice that the repair time distribution is such that $R(t) = 1$ for all $t > t_{max}$, a fixed value. That is, a finite upper bound on repair times exists. When this is the case,

$$\int_0^t (1-R(s))ds = \int_0^{t_{max}} (1-R(s))ds = E[T]$$

and $$P[X(t) = x] = \frac{e^{-p\lambda E[T]}(p\lambda E[T])^x}{x!} \quad \text{for all } t \geq t_{max}.$$

Finally, although the distribution $R(t)$ is called the repair time distribution, we actually are interested in the amount of time which elapses from the occurrence of a Type 1 failure until it is restored to serviceable condition and placed in the pool of spares. Thus, we are really interested in the distribution of $T_1 + T_2 + T_3$, where T_1 is the random amount of time required to transport the carcass of a Type 1 failure to the repair facility, T_2 is the actual time to repair the unit and T_3 is the time required to transport the unit from the repair facility to the pool of spares. Since the times T_1, T_2, and T_3 are independent we should redefine $R(t)$ to be the convolution of the distributions of T_1, T_2, and T_3. As Theorem 5.2 states, this is all really academic for all that we need is $E[T] = E[T_1] + E[T_2] + E[T_3]$.

6. THE DISTRIBUTION OF THE ON-HAND INVENTORY.

We are now ready to combine the results obtained in the last three sections to determine the stationary distribution of the on-hand inventory. We have shown that there is a random variable Y such that $Y(t)$ converges in distribution to Y as t gets large, denoted $Y(t)\xrightarrow{D}Y$. We have also shown that there exists a random variable X such that $X(t)\xrightarrow{D}X$. We will now prove that there is a random variable H such that $H(t)\xrightarrow{D}H$. Moreover, we will show that the distribution of H is given by the convolution of the distributions of Y and $-X$.

<u>Theorem 6.1</u>: If $Y(t)\xrightarrow{D}Y$ and $X(t)\xrightarrow{D}X$, then there exists a random variable H such that $H(t)\xrightarrow{D}H$, and the distribution of H is given by the convolution of the distributions of Y and $-X$.

Proof: Theorem 3.2 states that $X(t)$ and $Y(t)$ are independent for each $t > 0$. Also, by the hypothesis, $X(t)\xrightarrow{D}X$ and $Y(t)\xrightarrow{D}Y$. Let X^* and Y^* be independent random variables having the same marginal distributions as X and Y, respectively. Then $X(t) \xrightarrow{D} X^*$ and $Y(t) \xrightarrow{D} Y^*$, and Billingsley [8] shows that the joint distribution of $X(t)$ and $Y(t)$ converges in distribution to the joint distribution of X^* and Y^*, denoted

$$(X(t),\ Y(t)) \xrightarrow{D} (X^*,\ Y^*).$$

Billingsley [8] also shows that if $(X(t),\ Y(t)) \xrightarrow{D} (X^*,\ Y^*)$ and $\phi(x,y)$ is a continuous function then $\phi(X(t),\ Y(t)) \xrightarrow{D} \phi(X^*,\ Y^*)$. In particular, if $\phi(x,\ y) = y - x$, then $Y(t) - X(t) \xrightarrow{D} Y^* - X^*$. Since Y^* and X^* are independent the distribution of $Y^* - X^*$ is given by the convolution of the distributions of Y^* and $-X^*$. Furthermore, $H(t) = Y(t) - X(t)$ so that the above material shows that the distribution of $H(t)$ converges to the convolution of the distributions of Y^* and $-X^*$ which is the same as the convolution of the distributions of Y and $-X$. This completes the proof of the theorem.

We now combine the results given by Theorem 4.2, Theorem 5.2, and Theorem 6.1 to obtain the stationary distribution of the on-hand inventory.

<u>Theorem 6.2</u>: The stationary distribution of the on-hand inventory is given by

$$h(j) = \lim_{t\to\infty} P[H(t) = j] = \frac{1}{Q} \sum_{k=1}^{Q} f(r+k-j;\ \lambda_1 + \lambda_2),$$

where $\lambda_1 = p\lambda E[T]$, $\lambda_2 = (1-p)\lambda L$ and

$$f(u;\ \lambda_1 + \lambda_2) = \begin{cases} \dfrac{e^{-(\lambda_1 + \lambda_2)} (\lambda_1 + \lambda_2)^u}{u!} & \text{for } u \geq 0 \\ 0 & \text{for } u < 0. \end{cases}$$

Proof: By Theorems 4.2, 5.2, and 6.1 the stationary distribution of the on-hand inventory is given by the convolution of the distributions of

Y and -X. Thus,

$$h(j) = \sum_{x=0}^{\infty} \alpha(x+j) f(x;\lambda_1) \qquad \text{for} \quad j \le r + Q.$$

On substituting the expression given by Theorem 4.2 for $\alpha(x+j)$ we get

$$h(j) = \frac{1}{Q}\left[\sum_{x=0}^{r-j} \sum_{u=r+1-x-j}^{r+Q-x-j} f(u;\lambda_2) f(x;\lambda_1) + \sum_{x=r+1-j}^{r+Q-j} \sum_{u=0}^{r+Q-x-j} f(u;\lambda_2) f(x;\lambda_1)\right].$$

If we let $k = u - r + x + j$ and change the order of summation we obtain the simpler expression:

$$h(j) = \frac{1}{Q}\left[\sum_{k=1}^{Q} \sum_{x=0}^{r+k-j} f(r+k-j-x;\lambda_2) f(x;\lambda_1)\right].$$

On writing the expressions for $f(r+k-j-x;\lambda_2)$ and $f(x;\lambda_1)$ we get

$$h(j) = \frac{1}{Q}\left[\sum_{k=1}^{Q} \frac{e^{-(\lambda_1+\lambda_2)}}{(r+k-j)!}\left(\sum_{x=0}^{r+k-j} \binom{r+k-j}{x} . \lambda_1^{x} . \lambda_2^{r+k-j-x}\right)\right]$$

$$= \frac{1}{Q}\left[\sum_{k=1}^{Q} \frac{e^{-(\lambda_1+\lambda_2)} (\lambda_1+\lambda_2)^{r+k-j}}{(r+k-j)!}\right]$$

$$= \frac{1}{Q} \sum_{k=1}^{Q} f(r+k-j;\lambda_1+\lambda_2).$$

This completes the proof of Theorem 6.2.

We find that the stationary probability mass function for the on-hand inventory has a particularly simple form being a finite sum of Poisson probabilities. This simple form facilitates the calculation of the desired measures of performance.

7. MEASURES OF PERFORMANCE

It is natural to ask what the effects of a given policy would be. Most inventory studies attempt to evaluate policies or sets of parameters for a given policy by looking at the costs incurred. In such studies alternative A is considered preferable to alternative B if the total expected cost incurred from the use of alternative A is smaller than that experienced when using alternative B. If all of the cost elements can be determined, this procedure leaves little to be desired. The shortcoming of the procedure is that the determination of the cost elements is itself a very difficult problem. If one is not able to specify the cost functions, an alternative is to concentrate instead on determining the magnitudes of such measures of effectiveness as the expected number of backorder days, the probability of being out of stock, the expected number of backorders, mean supply response time, the expected on-hand stock and the operational availability. These expressions would then be surveyed to see if they are satisfactory.

The goal of every inventory system should be to provide maximum service subject to given resources or to provide maximum service at minimum cost. To provide maximum service, the inventory system attempts to protect itself against the possibility that a failed item cannot be replaced immediately from stock on hand. Since failures and repair times are random and procurement lead times are positive, it is impossible to guarantee that each failed item can be replaced without delay. The best that can be done is to guarantee that the probability of the foregoing occurrence is small or that the expected penalty does not exceed a given positive quantity.

A valuable measure of the effectiveness of a given inventory system is the probability that a failed unit cannot be replaced immediately from stock on hand. This probability is obviously dependent on the size of the spare pool, which in turn depends on the parameters r and Q. With increasingly larger spare pools this probability can be made arbitrarily close to zero. However, investment constraints place upper bounds on the size of the spare pool, and hence, also the values of r and Q.

We can determine the probability that a failed unit cannot be replaced immediately from stock on hand, denoted P_{out}, by a straightforward summation of probabilities of the on-hand inventory. A stockout will occur if and only if the stock on hand is less than or equal to zero when a demand occurs. Therefore, the probability of a stockout is given by

(7.1)
$$
\begin{aligned}
P_{out} = \sum_{j=0}^{\infty} h(-j) &= \frac{1}{Q} \sum_{j=0}^{\infty} \sum_{k=1}^{Q} f(r+k+j;\, {}_{\cdot}\lambda_1 + {}_{\cdot}\lambda_2) \\
&= \frac{1}{Q} \sum_{k=1}^{Q} \sum_{j=0}^{\infty} f(u;\, {}_{\cdot}\lambda_1 + {}_{\cdot}\lambda_2) \\
&= \frac{1}{Q} \sum_{k=1}^{Q} F(r+k) \\
&= \frac{1}{Q} \{({}_{\cdot}\lambda_1 + {}_{\cdot}\lambda_2)\,[F(r) - F(r+Q)] - rF(r+1) + (r+Q)\,F(r+Q+1)\},
\end{aligned}
$$

where
$$F(u) = \sum_{i=u}^{\infty} f(i;\, {}_{\cdot}\lambda_1 + {}_{\cdot}\lambda_2).$$

From the probability of a stockout, we easily obtain the expected number of stockouts per year, denoted $E(r,Q)$, by multiplying the average number of demands per year by the stationary probability of a stockout. That is,

(7.2)
$$E(r,Q) = {}_{\cdot}\lambda \cdot P_{out}.$$

A third measure of performance is concerned not only with the number of stockouts, but also with the amount of time required to satisfy the unfilled demands. This measure of performance, called the time-weighted backorders, weights each backorder by the amount of time the backorder is outstanding and sums over all backorders. One backorder unfilled for ten days is considered as undesirable as ten backorders unfilled for one day each. Although this may not be rigorously true, it is clearly important to consider the magnitude of the delays. Whenever the on-hand inventory has a stationary distribution the total time-weighted backorders incurred in a unit period of time is equivalent to the expected number of backorders at an arbitrary point in time. (See Hadley and Whitin [3].) Let $B(r,Q)$ denote the expected number of backorders at time t where t is large when the reorder quantity is Q and the reorder level is r. Then, for $j \geq 0$, there will be j backorders at time t if and only if the on-hand inventory is $-j$. A straightforward calculation yields

$$B(r,Q) = \sum_{j=0}^{\infty} jh(-j) = \sum_{j=0}^{\infty} j\left[\frac{1}{Q} \sum_{k=1}^{Q} f(r+k+j;\, \lambda_1 + \lambda_2)\right]$$

$$= \frac{1}{Q} \sum_{k=1}^{Q} \left[\sum_{u=r+k}^{\infty} (u-(r+k)) \quad f(u;\, \lambda_1 + \lambda_2) \right]$$

The expression on the right side of the equality can be simplified further so that $B(r,Q)$ is given by

$$B(r,Q) = \frac{1}{Q}\left\{\frac{(\lambda_1 + \lambda_2)^2}{2} [F(r-1) - F(r+Q-1)] - (\lambda_1 + \lambda_2)[rF(r) - (r+Q)F(r+Q)] + \frac{r(r+1)}{2} F(r+1) - \frac{(r+Q+1)\ (r+Q)}{2} F(r+Q+1)\right\}. \tag{7.3}$$

This gives $B(r,Q)$ with the dimensions of unit-years of backorders. To convert to somewhat more meaningful units simply multiply by 365 to get total expected backorder days per year.

Although it is desirable to provide a high level of service, a price must be paid for that capability. One of the costs previously mentioned is the investment cost of the spares. Another cost which must be considered is the inventory holding cost, or the cost paid to maintain the pool of spares. It includes both tangible costs such as warehouse costs, labor, theft and obsolescence and the intangible cost of having money tied up in idle resources. Since the holding cost at any time is roughly proportional to the dollar investment in spares, another useful measure of performance is the expected dollar investment of the ready-for-issue stock or the expected value of the ready-for-issue stock, denoted $D(r,Q)$. The net inventory is equal to the ready-for-issue stock minus backorders plus the repair inventory. By Theorem 4.2 the expected value of the net inventory is $r+\frac{Q+1}{2} - (1-p)\lambda L$ and the expected value of the repair inventory is $p\lambda E[T]$, so that

$$D(r,Q) = r+\frac{Q+1}{2} - (1-p)\lambda L + B(r,Q) - p\lambda E[T]. \tag{7.4}$$

A holding cost must also be paid for holding units at the repair facility. As with the ready-for-issue stock this holding cost is usually proportional to the average number of units held at the repair facility, which is $p\lambda E[T]$.

Finally, there is a cost incurred each time a procurement is made. This "setup cost" includes all of those costs which are not accounted for by the actual price of the units purchased. This cost, assumed to be independent of the number of units purchased, includes all of those administrative costs associated with placing an order including, primarily, the cost of letting a contract. The total expected setup cost per unit time is therefore proportional to the expected number of orders placed per year which is given by $(1-p)\lambda/Q$.

The measures of performance which we have determined are useful in many ways. If one were able to ascertain the appropriate cost functions, one could employ the measures of performance to determine the expected costs for a given set of parameters or even to determine the "least cost" pair of parameters. Examples of this approach are presented in a later section. If the determination of the cost functions is not feasible, the measures of performance given here enable a decision maker to accept or reject certain values of the decision variables or to compare a limited number of alternatives. Thus, they offer at least a partial substitute to a procedure which determines optimal solutions.

8. THE SPECIAL CASES $P = 0$ AND $P = 1$

If one considers the case $p = 0$, all failed items are lost to the system (there is no repair) and must be replaced by procurements. The inventory system then reduces to a consumable item system, and the results obtained here reduce to those well-known results in the literature concerning consumable items. In fact, for $0 < p < 1$, the reader familiar with that literature will note a great deal of resemblance between the results derived here and those previously known for the consumable item system with Poisson demands. That similarity is an interesting theoretical feature of this repairable item model.

If the value of p is taken to be unity, implying no attrition, a few modifications of the foregoing results must be made. First, the inventory position does not bounce between two levels as before, but remains constant. Let us denote the value of the inventory position by R. Because there is no attrition, the reorder level r and reorder quantity Q no longer have any meaning. We are interested, however, in examining various values of R, for R represents the total number of spares in the system.

An important decision, especially for initial provisioning of parts which are failure prone, is the size of the spares pool which is required to support a given component. We will derive measures of effectiveness which are useful in making that decision. First, we must derive the distributions of the various inventories.

To find the distribution of the on-hand inventory we note that the net inventory, like the inventory position, is constantly R, for there is no distinction between the two when $p = 1$. The on-hand inventory is h if and only if the repair inventory is $R - h$. Thus,

$$\lim_{t\to\infty} P[H(t)=h] = \lim_{t\to\infty} P[X(t)=R-h]$$

$$= \begin{cases} \dfrac{e^{-\lambda E[T]}(\lambda E[T])^{R-h}}{(R-h)} & \text{if } h \le R \\ 0 & \text{if } h > R \end{cases}$$

Straightforward calculations then give

(8.1)
$$P_{out} = F(R)$$
$$E(R) = \lambda F(R)$$
$$B(R) = \lambda E[T]\ F(R-1) - RF(R)$$
$$D(R) = R - \lambda E[T] + B(R)$$

and the expected number in repair is $\lambda E[T]$, where we write $E(R)$, $B(R)$, and $D(R)$ in place of $E(r,Q)$, $B(r,Q)$, and $D(r,Q)$, respectively, for obvious reasons.

9. NUMERICAL EXAMPLES AND DISCUSSION OF RESULTS

To demonstrate the uses of the results in Section 7 and Section 8 and to clarify the calculations we now present numerical examples, and we discuss some cost-tradeoff studies. We will present tables containing numerical values of various measures of supply performance for a range of values of the system parameters. First, however, we illustrate how the optimal values of the parameters are chosen if costs can be assigned. As is common, we assume that the setup cost is a constant, A, independent of the quantity purchased, the expected annual holding cost for ready-for-issue units is $H_1CD(r,Q)$ and for not-ready-for-issue units the expected annual holding cost is $H_2Cp\lambda E[T]$, where H_1 and H_2 are proportionality constants and C is the unit cost of the item. Each time a backorder occurs a fixed cost of size π is incurred. If the backorder is outstanding t units of time, an additional cost of magnitude $\pi' t$ is paid. The cost of acquiring Q units is CQ so that no discounts are earned by ordering large quantities. Since all Type 1 failures must be repaired and all Type 2 failures must be replaced at costs independent of the parameters r and Q, both repair costs and acquisition costs are ignored at this time. Thus, with our assumptions, the expected annual cost for $0 \leq p < 1$ is given by

$$(9.1)\quad K(r,Q) = \frac{\lambda}{Q} A + H_1CD(r,Q) + H_2Cp\lambda E[T] + \pi E(r,Q) + \pi' B(r,Q),$$

and for $p = 1$, the expected annual cost is

$$(9.2)\qquad K(R) = H_1CD(R) + H_2C\lambda E[T] + \pi E(R) + \pi' B(R).$$

As we have mentioned previously, costs are difficult to determine, especially π and π'. However, for completeness, we assume values for the costs and determine the pair (r^*,Q^*) and the singleton R^* which minimize $K(r,Q)$ and $K(R)$, respectively, for given values of the other parameters.

The cost equations (9.1) and (9.2) are complicated expressions of the parameters (r,Q) in the former and R in the latter. Attempts to determine explicit equations for r^*, Q^*, and R^* have been unsuccessful. In fact, even when $p = 0$, implying consumable units, explicit equations for these variables are unknown. In spite of these difficulties, it is not necessary to resort to sophisticated search techniques to determine the optimal values. This is because the repairable item model is most appropriate when demands are low and purchase costs are high. Thus, practical limits on the number of units that can be stocked economically are dictated. In addition r, Q, and R are integers, so only a small number of alternatives need be considered. Perhaps the easiest way to determine r^*, Q^*, and R^* is to calculate the costs for all feasible choices of the variables and pick those which minimize costs. Additionally, although it has never been proved analytically, there is overwhelming empirical evidence that the cost equations (9.1) and (9.2) are convex. This is also intuitively true for if the spare stock or safety stock is low expected costs will be high because of high stockout costs. As the stock is increased the expected costs should decrease until the spare stock is large enough so that the increase in holding costs exceeds the decrease in stockout costs. Then the expected costs continue rising as the stock is increased. This property reduces further the number of alternatives which need be considered. Table 1 gives the expected annual cost $K(r,Q)$ for a range of feasible values of r and Q with the parameters $A = \$100$, $H_1 = 0.20$, $H_2 = 0.18$, $C = \$2500$, $\pi = \$1000$, $\pi' = \$1000$, $L = 0.50$ years, $p = 0.85$, $\lambda = 16$ and $E[T] = 0.25$ years.

A look at Table 1 shows that $K(r,Q)$ is convex in r for each Q, and $K(r,Q)$ is convex in Q for each fixed value of r. Also, $r^* = 8$, $Q^* = 2$ and the minimum expected annual cost is $K(r^*,Q^*) = \$4650$. (This includes neither the acquisition cost, nor the repair cost.)

K(r,Q)										
Q \ r	3	4	5	6	7	8	9	10	11	12
1	13991	10760	8028	6139	5106	4739	4801	5101	5518	5986
2	12255	9274	6964	5502	4802	4650	4831	5189	5632	6111
3	10766	8149	6264	5168	4722	4720	4980	5375	5833	6317
4	9550	7328	5823	5016	4756	4859	5171	5590	6057	6546
5	8613	6762	5570	4985	4861	5037	5384	5818	6292	6783
6	7927	6395	5452	5034	5008	5237	5608	6053	6532	7024
7	7446	6176	5427	5136	5184	5450	5840	6293	6777	7268
8	7123	6064	5467	5273	5377	5672	6077	6535	7019	7513
9	6918	6029	5553	5435	5583	5901	6316	6779	7265	7759
10	6801	6049	5670	5614	5798	6134	6557	7024	7511	8006
11	6749	6111	5812	5807	6019	6370	6800	7270	7758	8254
12	6748	6204	5972	6009	6245	6608	7044	7517	8006	8502
13	6785	6322	6146	6218	6474	6849	7289	7764	8254	8750
14	6853	6458	6330	6433	6707	7090	7535	8012	8502	8999
15	6944	6610	6524	6653	6942	7333	7782	8259	8751	9248

TABLE 1

PARAMETER VALUES: $p = 0.85$ OPTIMAL PARAMETERS: $r^* = 8$

$\lambda = 16.0$ $Q^* = 2$

$E[T] = 0.25$ years

$L = 0.50$ years

COSTS: $A = \$100.$ $H2 = 0.18$

$C = \$2500.$ $\pi = \$1000.$

$H1 = 0.20$ $\pi' = \$1000.$

Let us now consider the case $p = 1$ and find the optimal number of spares, R^*. Let LTS represent the lifetime of the spares and let LTC depict the lifetime of the system or component for which the spares are provided. Then let LT = min [LTS,LTC]. If LT = LTS, let $I(R)$ be the investment in spares allocated over LTS years, and if LT = LTC let $I(R)$ be the procurement cost of R spares less the salvage value of the spares allocated over LTC years. For simplicity in this example we assume that LT = LTS, or equivalently that LT = LTC and the salvage value is zero. Using the same values of the parameters as in Table 1, except for the change in p, and LT = 5 years, we obtain the values presented in Table 2.

From Table 2 we observe that the optimal number of spares, that is, the value of R^* which minimizes $K(R) + I(R)$, is $R^* = 8$. If the lifetime of a spare were a single year, the optimal number of spares is easily found to be $R^* = 5$. This points out the importance of considering the lifetimes when determining the size of the rotatable pool of spares. Note also in Table 2 that $K(R)$ and, consequently, $K(R) + I(R)$ are convex.

As discussed previously, it is very difficult, if not impossible, to obtain meaningful values for the stockout costs. Even if the foregoing assumptions about the stockout costs are appropriate, the constants π and π' are extremely elusive. However, we can still rely on our measures of supply effectiveness to determine acceptable parameter values for R or r and Q. When presented with a set of alternatives an inventory manager can evaluate those alternatives and choose the one which maximizes his utility for the alternatives. The measures of effectiveness provide a basis for evaluating the alternatives. We illustrate this with an example.

R	K(R)	I(R)	K(R) + I(R)
2	18500.	1000.	19500.
3	15512	1500.	17012.
4	12037.	2000.	14037.
5	8854.	2500.	11354.
6	6531.	3000.	9531.
7	5198.	3500.	8698.
8	4669.	4000.	8669.
9	4660.	4500.	9160.
10	4936.	5000.	9936.
11	5347.	5500.	10847.
12	5815.	6000.	11815.

TABLE 2

PARAMETER VALUES: $p = 1.0$

$\lambda = 16.0$

$E\left[T\right] = 0.25$

OPTIMAL NUMBER OF SPARES: $R^* = 8.$

MINIMUM COST = \$8669.

Consider the same case as that summarized by Table 1, but now let us omit the stockout costs. Instead, for a range of values for r and Q, we determine the probability of a stockout, P_{out}; the total expected backorder days per year, BOD; and the total ordering and holding costs, COST. The values are displayed in Table 3. The inventory manager could search through Table 3 until he determines that pair (r,Q) which maximizes his utility. For example, he may decide that he wishes to minimize ordering and holding costs subject to the restriction that the probability of a stockout not exceed 0.05. For that objective, the best choices of r and Q are found to be $r^* = 8$, $Q^* = 1$. On the other hand, he may decide to consider a constraint on backorder days, or average investment in spares, or total budget or yet some other measure. The point we emphasize is that, whatever the decision criteria, decisions will probably be based on the measures of performance which we have determined.

We finish this section with a brief discussion of possible uses of the results to consider tradeoffs among the system parameters. There are ways to achieve specified levels of system performance other than through an increase in spare stock or safety stock. In particular, reductions in the mean repair time or the procurement lead time also reduce the probability of being out of stock and the expected number of backorder days. Thus, it may be possible to achieve acceptable levels of system performance with an accompanying reduction in spare stock, investment and holding costs by decreasing the lead times. It would behoove the inventory manager to consider cost tradeoffs between increasing stock and reducing the lead times. Perhaps lead time reductions can be accomplished by overtime utilization, increased workforces, expedited shipments or other methods which are more economical than increasing the stock. The studies of cost tradeoffs such as this are

MEASURES OF EFFECTIVENESS

Q \ r	4			5			6		
	P_{out}	BOD	COST	P_{out}	BOD	COST	P_{out}	BOD	COST
1	.487	243.8	2304	.314	129.1	2647	.182	62.7	3056
2	.401	186.4	2355	.248	95.9	2731	.139	45.3	3162
3	.328	145.2	2509	.197	73.2	2910	.107	34.1	3357
4	.270	115.9	2699	.159	57.8	3119	.085	26.6	3576
5	.225	95.0	2908	.131	47.1	3343	.070	21.6	3808
6	.190	79.9	3129	.111	39.5	3574	.059	18.1	4045
7	.164	68.7	3358	.095	34.0	3811	.050	15.5	4286
8	.144	60.2	3592	.083	29.7	4051	.044	13.6	4529
Q \ r	7			8			9		
	P_{out}	BOD	COST	P_{out}	BOD	COST	P_{out}	BOD	COST
1	.095	28.0	3508	.045	11.5	3986	.020	4.4	4476
2	.070	19.8	3627	.032	8.0	4111	.014	3.0	4604
3	.053	14.6	3830	.024	5.8	4318	.010	2.2	4813
4	.042	11.4	4056	.019	4.5	4546	.008	1.7	5042
5	.034	9.2	4291	.015	3.6	4783	.006	1.3	5280
6	.029	7.7	4531	.013	3.0	5024	.005	1.3	5522
7	.025	6.6	4773	.011	2.8	5268	.005	0.9	5766
8	.021	5.9	5018	.010	2.2	5513	.004	0.8	6011

TABLE 3

apt to make important contributions to the problems of resource allocation in supply systems.

10. CONCLUSIONS AND RECOMMENDATIONS

Expressions for measures of supply performance have been derived for a probabilistic model of a repairable item inventory system. As with any mathematical model of a real world situation, some assumptions have been made which may not be rigorously true. Besides the assumption about the distribution generating demands, the only critical assumption is the one which states that there are ample servers so that a unit never has to wait at the repair facility before repairs begin. For the items of interest, the number of failures per unit time is usually sufficiently small and repair times are sufficiently short so that the maximum number of servers required at any time is not large. On the other hand, if the repair facility supports many different types of items, it may be that a carcass must experience a delay before it undergoes repair. In this case, perhaps some modification of the repair time distribution or the mean repair time might be appropriate. Further study would be necessary to determine what corrective actions are needed.

For some items repair setup costs are so large that it is economical to batch units for repair. This presents complicated mathematical problems in determining stationary distributions of the various inventories and measures of effectiveness. A probabilistic model of a repairable item inventory system which allows for batching of units for repair will be examined in a later report.

Although the model we consider in this report looks at a single item it is of substantial use in examining complex multi-item military inventory systems. For in such a complex system often the only interaction among the items is through their competition for the resources in the system. Thus, measures of performance for each of the items can be determined independently of the other items, and these measures can be used to determine how resources should be allocated among the items to maximize system effectiveness. A case can also be made to support the contention that it would be appropriate to concentrate individual attention on particular items, for some repairable items are sufficiently expensive and essential to justify exclusive management attention.

Many applications of the model for determining the optimal parameters for the operation of the inventory system, initial provisioning and cost-tradeoff studies were discussed and illustrated in the last section. The model can also be used to evaluate tradeoffs between increasing the value p or increasing the reliability of the components and increasing spare stock. Furthermore, the model should be an aid in determining whether or not an item is to be designated as repairable or consumable.

The results obtained in this study do not offer a complete solution to the complex problems facing the inventory manager of a repairable item inventory system, but they do supply him with tools to use in making decisions. Indeed, a major objective of this study is simply to highlight the importance of the use of the various measures of supply performance in making intelligent decisions about operating policies and resource allocation.

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13. ABSTRACT

An inventory system for repairable items is studied. A stochastic model which coordinates procurement and repair decisions is developed. Special attention is paid to modeling the repairable item system so that the results derived in this report are applicable for Navy inventory management. Long-run distributions for both the ready-for-issue and the non-ready-for-issue stock and many useful measures of performance are determined. Uses of the information to analyze the critical factors in a repairable item system and to determine optimal values of the parameters are pointed out. Numerical examples of the calculations of the measures of effectiveness are presented. Finally, modifications of the model to include the zero attrition case and consumable items are pointed out.

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